Name—————————————, ID ——————

Exam One , MTH 221 , Fall 2019

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SCORE = $\frac{}{54}$

QUESTION 1. (10 points) Imagine the following setting:
You are "Mr/Ms know it". Your friend just sent you the following messages on "WhatsApp"

(i) Hi. My instructor "Badawi", "Bawadi", mmmm, not sure about the name, today he was talking about independent. My question: What does it mean that Q_1, Q_2, Q_3 are independent points in R^4?

Answer $a_1Q_1 + a_2Q_2 + a_3Q_3 = (0, 0, 0, 0)$ **for some real numbers** a_1, a_2, a_3 **implies** $a_1 = a_2 = a_3 = 0$

(ii) Thanks, one more please: What does it mean that 2019 is an eginenvalue of a matrix A, 5×5?

Answer **There is a nonzero point in R^5, say Q, such that** $AQ^T = 2019Q^T$

(iii) You are really good, "appreciate it", If a system of linear equations is not homogeneous, "Balawi" told us that the solution set cannot be written as Span of some points, mmmm, Why?

Answer **Since the system is not homogeneous, one of the equations has the form $a_1x_1 + ... + a_nx_n = c_n \neq 0$. Thus (0, 0, ..., 0) is not in the solution set of the system. Hence by class notes, the solution set can not be a subspace.**

(iv) WAW, you are amazing!, wish you are my teacher instead of my instructor, I know if Q_1, Q_2, Q_3, Q_4 are nonzero orthogonal points in R^5, then they are independent and the dot product of every two points of them is Zero. He mentioned that there is a "geometric" meaning for the nonzero points $Q_1, ..., Q_4$ to be orthogonal. He was "mumbling", did not understand him. Can you please tell me what is the "geometric" meaning for the nonzero points $Q_1, ..., Q_4$ to be orthogonal?

Answer (**imagine that we constructed the line segments OQ_1, OQ_2, ... , OQ_4, then the angle between every two line segments is 90 degrees**)

(v) You are genius, "WAW", Please PLEASE ONE MORE QUESTION

You: OK, last one, after that I am turning off my mobile.

How can I form a basis for R^6?

Answer **Choose 6 independent points in** R^6

(vi) Hello, ...Hello,..., Damn, he really meant it. He turned off his mobile. Damn, I still have two more questions.

QUESTION 2. (6 points) Let $F = \{(a_1 + a_2 + a_4,\ a_3 - a_4,\ a_1 + a_2 + a_3,\ -2a_3 + 2a_4) \mid a_1, a_2, a_3, a_4 \in R\}$.

(i) Convince me that F is a subspace of R^4. $F = \{a_1(1, 0, 1, 0)+a_2(1, 0, 1, 0)+a_3(0, 1, 1, -2)\} = span\{(1, 0, 1, 0), (0, 1$
Hence F is a subspace of R^4. now by using the Technique discussed in class, we see that IN(F) = dim(F) = 2.

(ii) Write F as span of an orthogonal basis (If the basis has more than 2 points, then just write down $W_3 = ...$, $W_4 =$ ...., and so on).

Clearly $F = \{W_1, W_2\}$, use gram-Schmidt algorithm to find $W_1, W_2, ...$, nothing fancy

QUESTION 3. (4 points)

(i) Convince me that $L = \{(a_1, a_2, -2a_2) \mid a_1 + a_2 = 1$, where $a_1, a_2 \in R\}$ is not a subspace of R^3 by showing that one of the subspace-axioms fails

Since 1 + 0 = 1, $(1, 0, 0) \in L$. **Now** $3(1, 0, 0) = (3, 0, 0) \notin L$ **because** $3 + 0 \neq 1$ (second axiom fails)

(ii) Convince me that $L = \{(a_1, a_2, a_1a_2) \mid a_1, a_2 \in R\}$ is not a subspace of R^3 by showing that one of the subspace-axioms fails.

$v = (1, 1, 1), w = (-1, -1, 1) \in L$. **But** $v + w = (0, 0, 2) \notin L$ **(first axiom fails)**

QUESTION 4. (8 points) Let $T : R^4 \to R^3$ such that $T(a_1, a_2, a_3, a_4) = (a_1 + 2a_3,\ -2a_1 - 4a_3,\ -a_1 - 2a_3)$ be a linear transformation.

(i) Write Range(T) as span of some independent points in R^3.

normal question, nothing fancy

(ii) Does the point $(3, -4, -2)$ belong to Range(T)? Explain

normal question, nothing fancy

(iii) Write Zeros of T, i.e., Z(T) as span of some independent points in R^4. (note, other authors refer to Z(T) as Ker(T) or null space of T)

normal question, nothing fancy

QUESTION 5. (4 points) Let $M = \{Q \in R^3 \mid Q \perp (1, -2, 1) \text{ and } Q \perp (0, 1, 3)\}$. Convince me that M is a subspace of R^3 by showing that M is a span of some independent points in R^3

$M = \{(x, y, z) \in R^3 \mid x - 2y + z = 0$ **and** $y + 3z = 0\}$. **Hence M is the solution set of the homogeneous system**

$$x - 2y + z = 0$$
$$y + 3z = 0$$

.

See class notes and write M as span of some independent points.

QUESTION 6. (6 points)

(i) Given 5 is an eigenvalue of the matrix $A = \begin{bmatrix} 4 & -1 & -1 & -1 \\ 1 & 6 & 1 & 1 \\ 2 & 2 & 6 & 3 \\ -1 & -1 & -1 & 4 \end{bmatrix}$. Write E_5 as span of some independent points.

Write the solution set of $(5I_4 - A) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

(ii) Let $F = 3I_4 + 2A^3$. Find a nonzero point Q in R^4 and a real number a such $FQ^T = aQ^T$.

Choose a nonzero point, say Q, in E_5. Then $FQ^T = [3 + 2(5)^3]Q^T$ = 253Q^T

QUESTION 7. (4 points) Let $A = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 6 \\ -3 & 6 & 9 \end{bmatrix}$. Find a matrix F, 3×10, such that Rank(F) = 2 and

$AF = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. SHOW the work[Hint: Think with minimum calculations]

Idea: note that each column of F lives in the solution set of the homogeneous system $\begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 6 \\ -3 & 6 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. Say the solution set is $S = span\{W, H\}$ (W, H are independent). Hence ONE way to construct F , $F = W^T|H^T|W^T|....|W^T$. By construction, $Rank(F) = 2$.

QUESTION 8. (6 points) Find the solution set of the following system (Note that the solution set is a subset of R^5)

$x_2 + x_3 - x_4 + x_5 = 1, \quad x_1 - x_2 + 4x_3 + 2x_4 - 4x_5 = -1, \quad 3x_1 + 2x_2 + 2x_3 - 2x_4 + 2x_5 = 2$

Nothing Fancy, just do it, form the augmented matrix, and do the calculations

Can you write the solution set of the above system as span of some points? EXPLAIN

No, it is not homogeneous

QUESTION 9. (3 points) Let A be a 3×5 matrix such that Rank(A) = 2. Convince me that there must be a point $Q = (a_1, a_2, a_3)$ in R^3 such that the system of linear equations $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ has no solution (i.e., inconsistent)[You do not need to find a_1, a_2, a_3]

Rank(A) = 2 implies only 2 columns in A, say F, L, are independent. Note that each column of A lives inside R^3. Hence, there is a point in R^3, say $M = (a_1, a_2, a_3)$, such that M is not a linear combination of F, L. Thus $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ **has no solution**

QUESTION 10. (3 points) Find a basis for the column space of A, where $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -2 & -2 & -1 & 8 & 4 \\ -3 & -3 & -3 & -3 & -2 \end{bmatrix}$.

NOTHING FANCY, TYPICAL CLASS NOTES QUESTION

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